STEM gems

Brought to you by the **NATIONAL AFTERSCHOOL ASSOCIATION**

A FIRE, THREE LIONS, THREE ZEBRAS AND A RAFT

big IDEAS

This STEM activity is short, easy to implement and requires only pen and paper. **THE FOCUS OF THIS ACTIVITY IS A TYPE OF MATH RIDDLE THAT HAS BEEN AMUSING THINKERS SINCE MEDIEVAL TIMES.** This type of problem can be fun and engaging and the key to successfully facilitating it is not to provide the answer too quickly. The value in this type of activity is giving young people plenty of time to think through the problem and design a solution. Young people will learn as much and have a deeper sense of accomplishment when struggling to find a solution and working through dead ends as they will finding the solution. When conducting this activity, assure young people there is no quick answer or tricks. This type of problem requires logically thinking through all the possible choices and their associated consequences, hence strengthening training and practice in both logical and critical thinking.

RIVER-CROSSING PROBLEM

River-crossing problems are fun, recreational math puzzles that have existed for thousands of years. These problems can take many different forms, but they all involve transporting items from one riverbank to another, usually in the fewest number of trips and following a set of restrictions, such as the number of items that can be transported at the same time or the number of items that may be safely left together. The following is an outline of a well-known river-crossing problem. A farmer must transport a wolf, goose and bag of beans from one side of a river to another using a boat, which can only hold one item in addition to the farmer. For obvious reasons, the wolf cannot be left alone with the goose and the goose cannot be left alone with the beans. These types of problems require thinking about all of the possibilities and eliminating the ones that don't work. The key is to think about all the choices that can be made at each point and the consequences of each choice. These types of problems can be solved by a branch of mathematics known as graph theory.

### HOW LONG HAVE PEOPLE BEEN SOLVING MATH RIDDLES?

People have been amusing themselves with math and logic problems for thousands of years. The Roman emperor Charlemagne loved puzzles and around 800AD he commissioned an English scholar, Alcuin of York, to create a book of puzzles for his amusement. The Propositiones ad Acuendos Juvenes, or Problems to Sharpen the Young, contains the first known occurrence of a river-crossing problem. These problems were the highlight of mathematics in medieval Europe and provide the focus of this STEM Gem activity.

Graph theory is the study of graphs, which are mathematical structures used to model relations between objects. A graph is a pictorial representation of a set of objects where some of the objects are connected by links. The interconnected objects are represented by points called vertices and the links that connect the vertices are called edges. In the wolf, goose and bag of beans puzzle, a graph can be drawn that accounts for all the possible combinations of each step. A dead end is created when a lethal state is reached. In this case, the wolf has eaten the goose or the goose has eaten the beans. A junction for more acceptable paths is created when a safe state is reached. The correct possible paths quickly become apparent when using this approach. See the graph on the right that illustrates a solution to the wolf, goose, and bag of beans puzzle.

Farmer= F; **Goose=**G; **Wolf=**W; **Beans=**B
Parentheses on the right = Finishing Bank
Parentheses on the left = Starting Bank
Blue = safe state; **Red =** lethal state
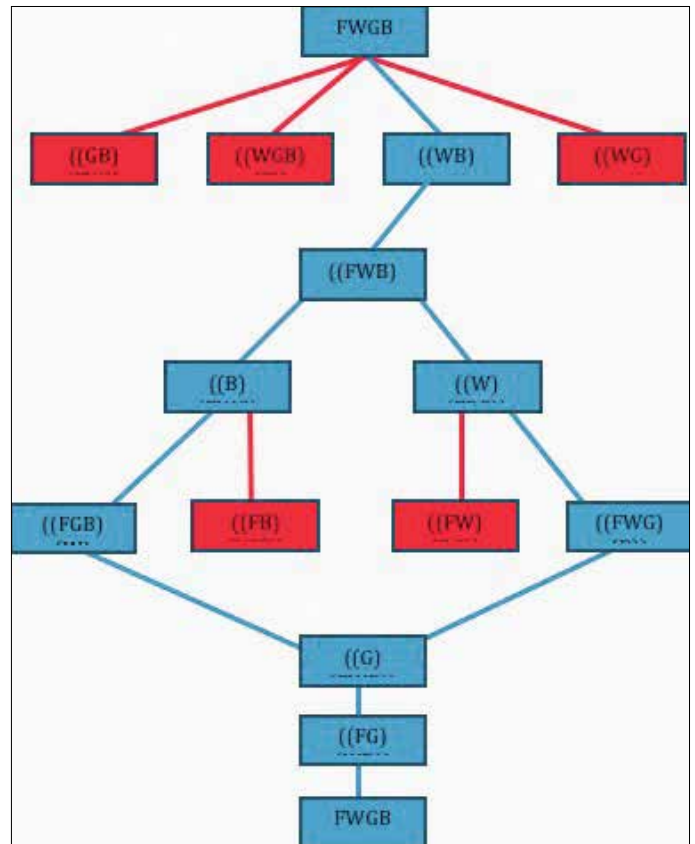
SCIENCE talk

GRAPH THEORY
A study of graphs

LOGIC
Ordered reasoning

engage

? **What is a riddle?** *An intentionally phrased question that requires ingenuity and critical thinking to answer. Riddles are typically presented as a game.*

? **Ask young people to share a riddle they are familiar with.** *Young people's choice*

? **Ask young people to share a math riddle they are familiar with.** *Young people's choice. An example includes: Which three numbers have the same answer either added together or multiplied together? Answer: 1, 2, and 3*

what YOU WILL NEED & before YOU BEGIN

Print out one set of three lions and three zebras for each small group working on the problem.

FOR EACH CHILD:

! **Writing tools**

! **Paper**

FOR EACH GROUP: *(optional manipulatives)*

! **One sheet of blue construction paper**

! **One set of three lions and three zebras printout**

! **One sheet of brown construction paper**

! **Scissors**

! **Colored pencils or markers**

① **Explain that in medieval times it was quite fashionable to set basic math problems as riddles.** These types of problems were very popular at parties and the lords and ladies present would try to solve them for fun. Tell young people that they will have an opportunity to try to solve one of these medieval math problems.

② **Divide young people in to small groups.** If using the manipulatives, provide each small group with one sheet of blue construction paper to represent the river, one sheet of brown construction paper to act as a raft, and one set of three lions and three zebras. Invite young people to decorate the lions and zebras and cut out each.

③ **Using the appropriate theatrics**, share the following problem with young people.

A brush fire is raging through the savannah grasslands of Africa. Three lions and three zebras are fleeing to escape the inferno. The animals have to cross a crocodile infested river to get to safety. Fortunately, there is a small raft that can keep them safe from the crocodiles. However, the raft can only carry two animals at a time and needs at least one animal to paddle it across the river. At no time can the lions outnumber the zebras, including when the animals are on the raft or on either side of the river. If the lions ever outnumber the zebras, their instincts will kick in and the zebra will become part of their menu. What is the fastest way, or least number of raft trips, for all the animals to escape the fire without the lions stopping for lunch?

RULES

1. **The raft needs at least one animal to paddle it across the river.**
2. **The raft can only hold one or two animals.**
3. **A zebra will be eaten if the lions ever outnumber the zebras on either side of the riverbank or on the raft.**
4. **The animals cannot swim across, but have to use the raft as described.**

④ **Invite the small groups to consider and discuss possible approaches to finding a solution.** Encourage the groups to use the manipulatives or paper and writing tools to map out all the possible solutions. The goal of this activity is for young people to not only discover the answer, but also discover the ordered way of thinking that they must use to find the answer. These types of problems have too many variables for young people to simply solve in their heads. It is best to find some way of modeling the problem and listing out the different choices and consequences.

⑤ **Allow adequate time for each group to work together trying different ideas and methods to solve the problem.** Provide encouragement and ask open-ended questions to help prompt their discoveries. If the groups need encouragement after some time has passed, provide a clue by telling young people that they need to consider all the choices that can be made at one point and to figure out the consequence of each choice. If the groups are struggling to get started, share with young people there are five possible choices to make the first crossing. Ask young people to share their ideas for these five possible choices as well as the result of each choice. Use the table below to discuss the first five choices and their associated consequence.

CHOICE	CONSEQUENCE
Send one lion across	If one animal crosses the river it will need to return, as the raft always needs an animal to paddle it.
Send one zebra across	If one animal crosses the river it will need to return, as the raft always needs an animal to paddle it.
Send two zebras across	If two zebras leave the riverbank, the one left with the three lions will immediately become lunch.
Send two lions across	Safe option that leads to a possible solution.
Send one lion and one zebra across	Safe option that leads to a possible solution.

6 **When each group has an answer to the riddle, invite them to demonstrate their answers for the other groups to check their work.** Share with young people that the fastest way to get all of the animals across the river is in 11 trips. There are two ways to do this. The answer and a pictorial summary are provided at the end of this activity.

make
THE CONNECTION

HAVE YOUNG PEOPLE USE LIBRARY BOOKS OR THE INTERNET TO RESEARCH AND TRY OTHER TYPES OF AGE-APPROPRIATE MATH RIDDLES. After they have practiced solving some math riddles, challenge them to create their own riddles to try out on one another.

EXTEND & EVALUATE

After they have solved the problem, ask young people if there is more than one solution to the lion, zebra, and raft problem. Explain to young people that there are two solutions. One solution involves a zebra and lion crossing the river first and the second solution involves two lions crossing first. Challenge young people to find the second solution to the problem. Both solutions take 11 steps. Ask young people which animal did the most rowing and if the work was equally divided. Invite young people to create a graph that illustrates their solution for others to follow. Encourage them to use their own ideas for creating the graph or guide them to draw a story of the problem like the pictorial summary shown at the end of the activity or a decision tree like the example shown in the Big Ideas section.

THE SHORTEST NUMBER OF TRIPS NEEDED TO GET ALL THE ANIMALS TO SAFETY IS 11. There are two paths to get to this answer. Path one involves sending two lions first, while the second successful path involves sending a lion and zebra first. If a lion and zebra are sent first, the result is two zebras and two lions on one side of the riverbank and one lion and one zebra on the opposite side of the riverbank.
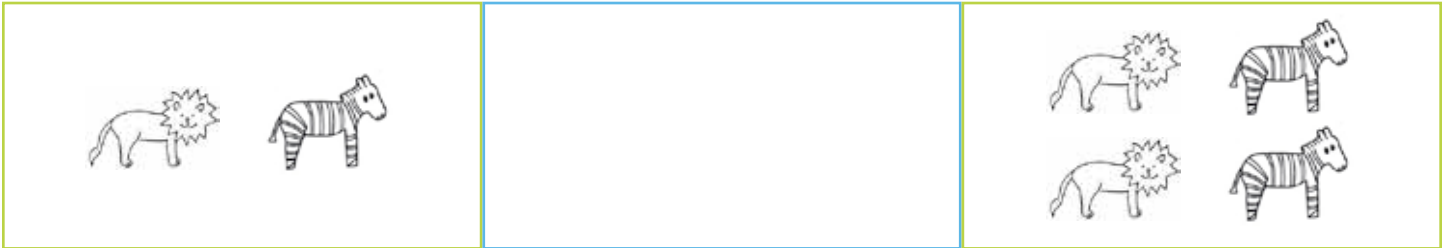

If the raft is sent back with a lion for the second crossing, the lions will outnumber the zebras on the right bank and they will have lunch. The only choice is to send the zebra back for **CROSSING NUMBER TWO**.
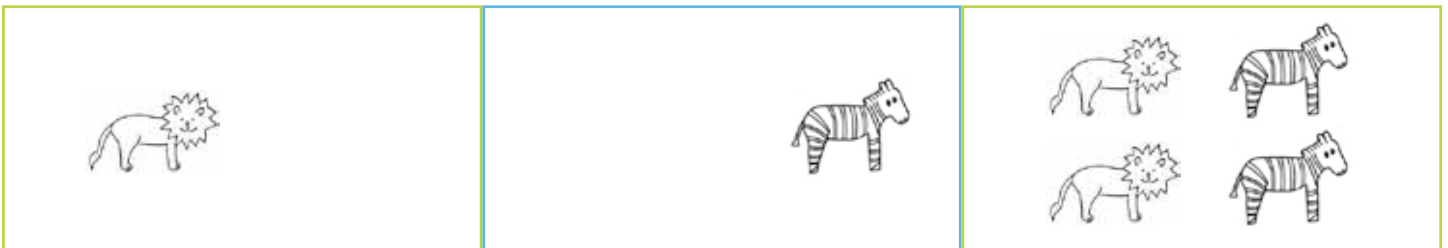
The same five choices as at the start are available for the **THIRD CROSSING**, but with one lion already on the left bank.

| CHOICE	CONSEQUENCE
Send one lion across	The lion will need to return, as the raft always needs an animal to paddle it.
Send a zebra and lion across	The lions will outnumber the zebras on the left bank and have lunch.
Send two zebras across	The lions will outnumber the zebras on the right bank and have lunch.
Send two lions across	Safe option that leads to a possible solution.
Send one zebra across	Same scenario as when started.
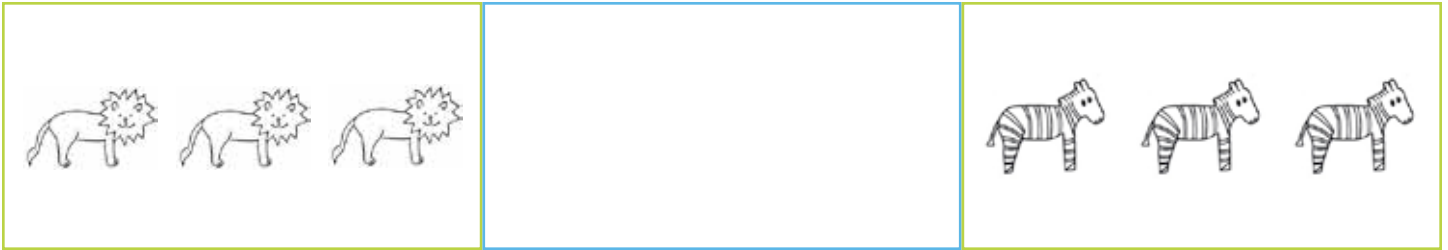
There are now three lions on one bank and three zebras on the opposite bank after the third crossing. One lion must be sent back for **CROSSING FOUR** so as not to repeat the last crossing.
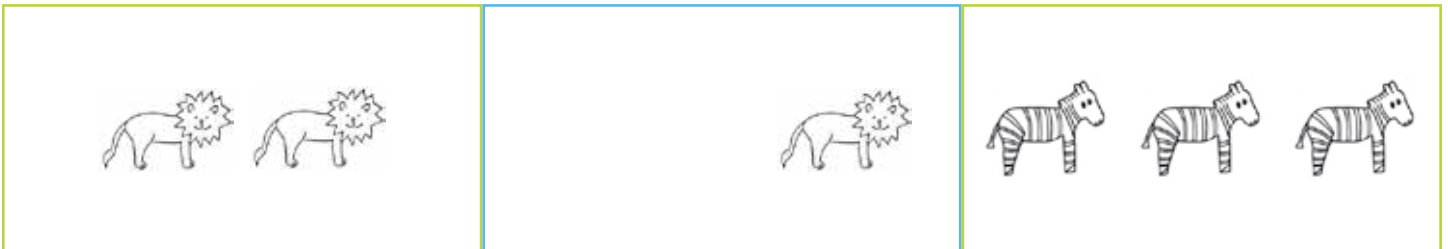

The only safe option for **CROSSING FIVE** is to send two zebras across to balance the two lions waiting on the left bank.
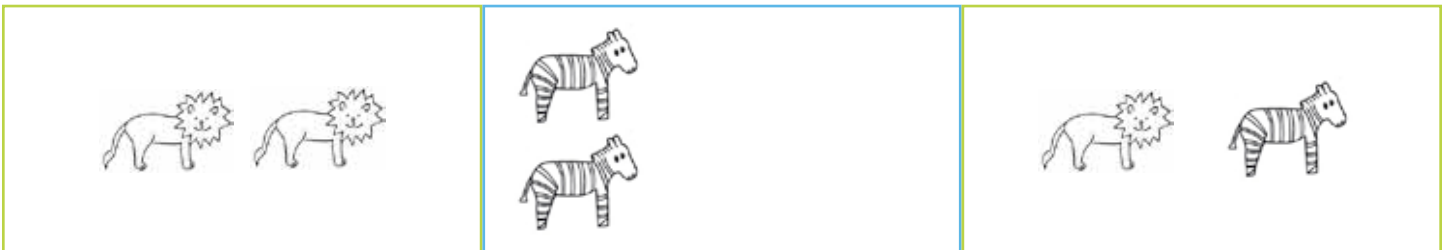
Two lions cannot be sent back to the right bank as they will outnumber the lonely zebra, so one lion and one zebra will need to be sent for **CROSSING SIX**. This step is the crux of the problem for successful completion as it seems as if you are going backward.
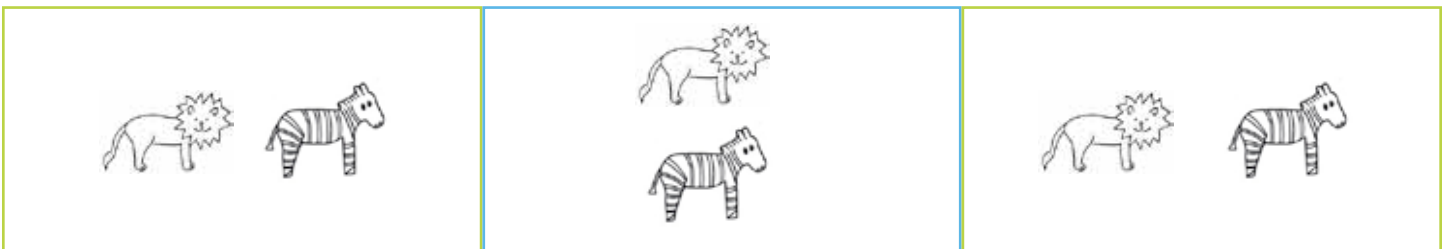
For **CROSSING SEVEN**, if a lion and zebra are sent back across it will be repeating the previous step. Sending two lions means the lions will outnumber the zebras on the left bank. The only safe option for crossing seven is to send the two zebras across.
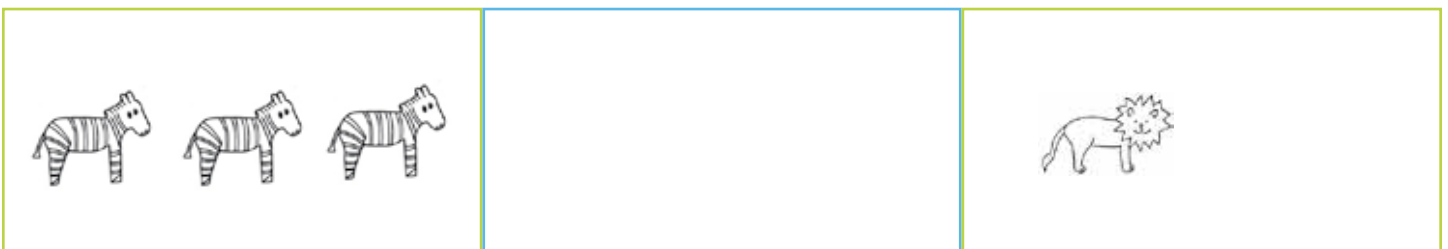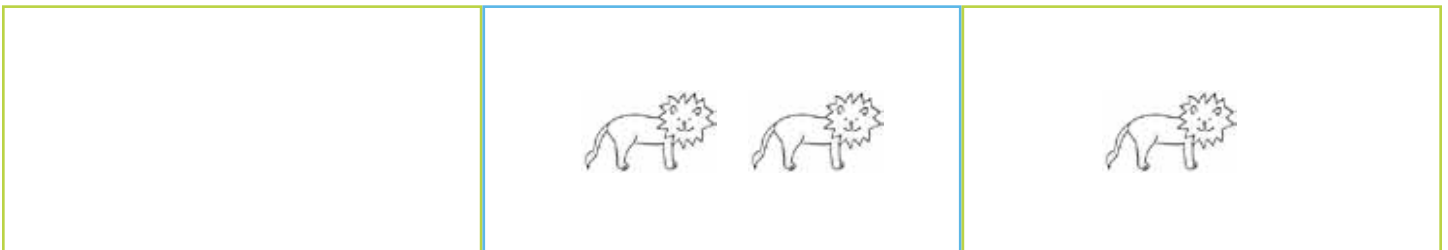
All of the zebras are safe on the left bank. The lions just have to make four more crossings to all get to safety.
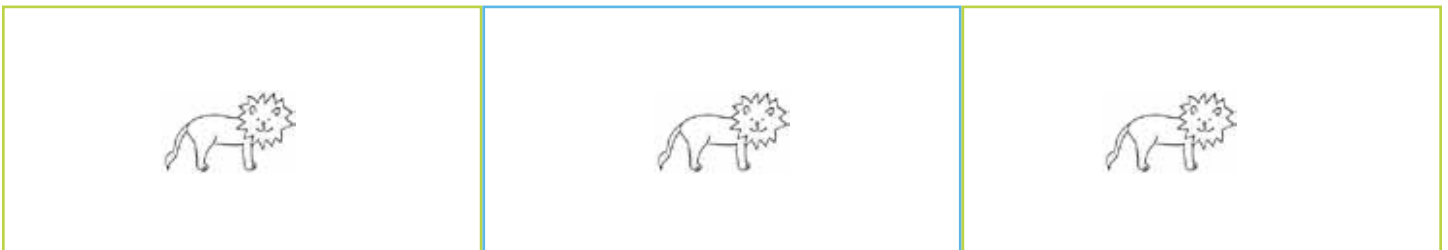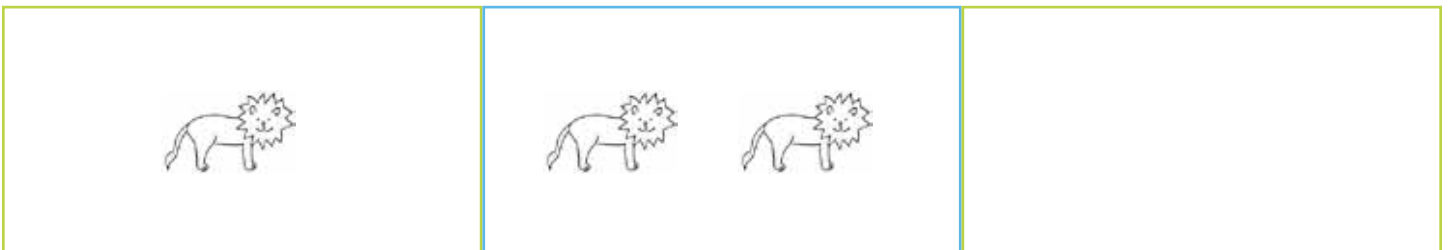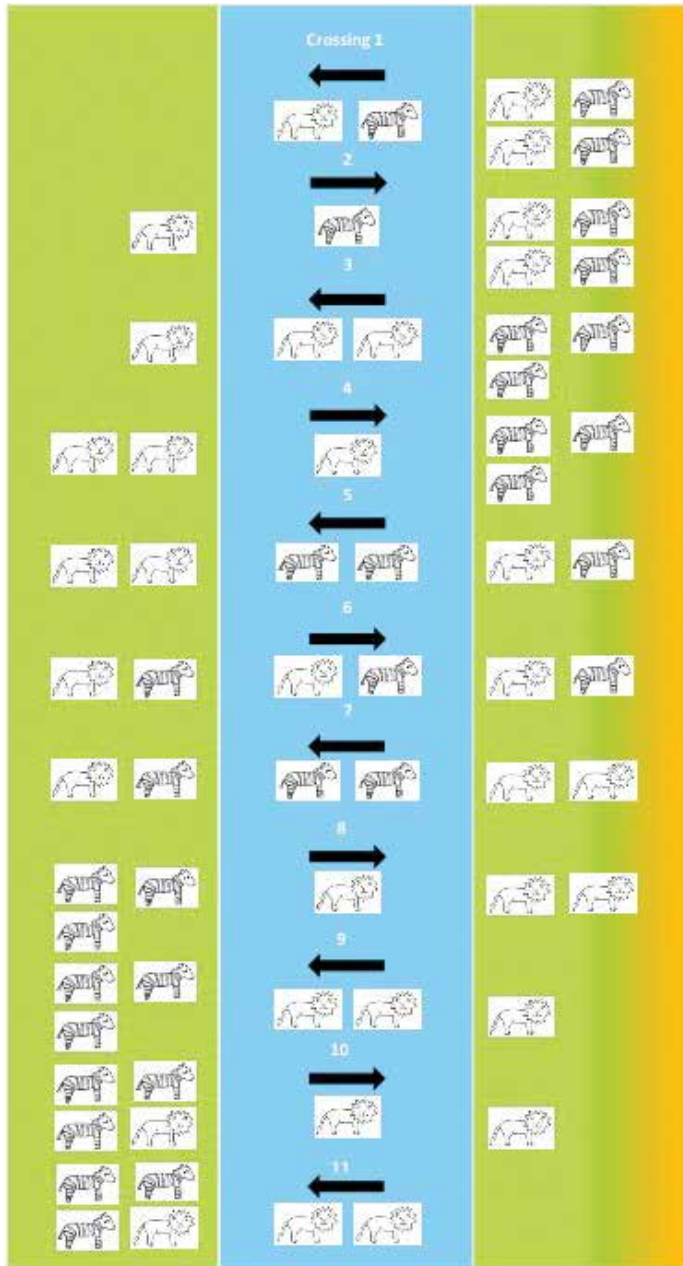
CROSSING EIGHT
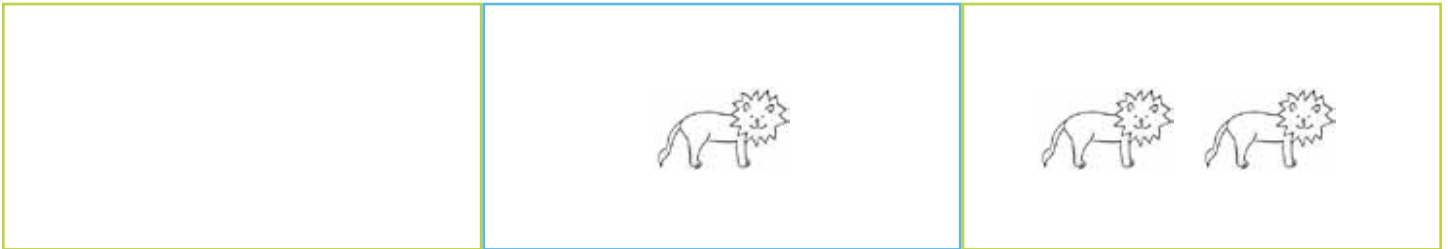
CROSSING NINE

CROSSING TEN

CROSSING ELEVEN

Finally, all zebras and lions are safe and sound.

SOLUTION 1 THE LION AND ZEBRA CROSS FIRST

SOLUTION 2 TWO LIONS CROSS FIRST